8. Powder mixing and blending;

Fluid flow through particle beds

8. Product tailoring by particle mixing,
   8.1 Microprocesses and mixing efficiency of particles
      8.1.1 Model of stochastic homogeneity
      8.1.2 Mixing kinetics
   8.2 Rotating vessels, kneaders and agitators
   8.3 Pneumatic mixing
      8.3.1 Permeation of particle beds
      8.3.2 Fluidized bed mixer
Processes or unit operations of mechanical process engineering according to RUMPF

<table>
<thead>
<tr>
<th>separation</th>
<th>combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>without change of particle size</td>
<td>powder mixing and blending</td>
</tr>
<tr>
<td>mechanical separation (filters, separators, screens, sifters)</td>
<td>particle size analysis</td>
</tr>
<tr>
<td>with change of particle size</td>
<td>size reduction (crushing and grinding)</td>
</tr>
<tr>
<td></td>
<td>size enlargement (agglomeration)</td>
</tr>
<tr>
<td>transport and storage of bulk materials</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 8.2 Prof. Dr. J. Tomas, chair of Mechanical Process Engineering
Characterisation of particle mixtures

1. characterisation of mixing states of a granular material or particle system

   a) completely separated

   b) ideal mixture
      (regular distribution)

   c) random mixture
2. Characterisation of the composition of a mixture and its quality of mixing

a) expected value $\mu_p = \frac{m_p}{m}$ of a composition of a mixture is known

$$\tilde{s}^2 = \frac{1}{n} \cdot \sum_{j=1}^{n} (\mu_{p,j} - \mu_p)^2$$

$\tilde{s}$ standard deviation, $\tilde{s}^2$ estimated variance of the expected value

$n$ number of samples

$\mu_{p,j}$ mass fraction of the component $p$ in the $j^{th}$ sample

a) expected value of a composition of a mixture is not known and has to be estimated

$$\overline{\mu}_p = \frac{1}{n} \cdot \sum_{j=1}^{n} \mu_{p,j}$$

(mean value (estimated expectation of $\mu_p$))

$$\tilde{s}^2 = \frac{1}{n - 1} \cdot \sum_{j=1}^{n} (\mu_{p,j} - \overline{\mu}_p)^2$$

3. Variance $\sigma_Z^2$ of stochastic homogeneity or an ideal random mixture, consisting of two components according to STANGE

$$\sigma_Z^2 = \frac{\mu_p \cdot \mu_q}{m_S} \cdot \left[ \mu_p \cdot \overline{m}_q \cdot (1 + v_q^2) + \mu_q \cdot \overline{m}_p \cdot (1 + v_p^2) \right]$$

$\mu_p$, $\mu_q$ mass fractions of the components (P) and (Q)

$m_S$ mass of a single sample

$\overline{m}_p$, $\overline{m}_q$ mean particle mass of the components (P) and (Q)
\[ V_p = \frac{\sigma_p}{m_p}, \quad V_q = \frac{\sigma_q}{m_q} \]

variation coefficients of the particle mass (size)
distributions of the components (P) and (Q)

4. Special cases:

a) narrow size distributions of both components \( v_p \to 0, v_q \to 0, \)

average particle number in one sample \( N_p = m_s / \bar{m}_p \)

\[ \sigma_Z^2 \approx \mu_p \cdot \mu_q / N_p \]

b) nearly equivalent size distributions \( \bar{m}_p \approx \bar{m}_q, \quad v_p \approx v_q \)

\[ \sigma_Z^2 \approx \frac{\mu_p \cdot \mu_q}{m_s} \cdot \bar{m}_q \cdot (1 + v_q^2) \]

c) large mass fraction of component (P) \( \mu_p > 0.9 \) and \( \bar{m}_p \approx \bar{m}_q \)

\[ \sigma_Z^2 \approx \frac{\mu_p \cdot \mu_q}{m_s} \cdot \bar{m}_q \cdot (1 + v_q^2) \]

d) component (P) is very coarse \( \bar{m}_p \gg \bar{m}_q \)

\[ \sigma_Z^2 \approx \frac{\mu_p \cdot \mu_q}{m_s} \cdot \bar{m}_q \cdot (1 + v_p^2) \]

5. Mixing efficiency \( ME = 1 - SE \) segregation index:

\[ ME_1 = \sigma_Z / \tilde{s} \quad \Rightarrow \sigma_Z / (\tilde{s} = \sigma_{max} \ldots \sigma_Z) = 0 \ldots 1 \]

\[ \sigma_{max} = \mu_p \cdot (1 - \mu_p) = \mu_p \cdot \mu_q \]

\[ ME_3 = 1 - \tilde{s} / \sigma_{max} \quad \Rightarrow 1 - (\tilde{s} = \sigma_{max} \ldots \sigma_Z) / \sigma_{max} = 0 \ldots 1 \]
### Process principles for input of mechanical energy into particulate systems

<table>
<thead>
<tr>
<th>Process principle</th>
<th>Schematic sketch</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>rotating process chambers or vessels</td>
<td><img src="" alt="Diagram" /></td>
<td>drum mixer&lt;br&gt;drum mill&lt;br&gt;drum dryer</td>
</tr>
<tr>
<td>agitated systems</td>
<td><img src="" alt="Diagram" /></td>
<td>mechanical silo&lt;br&gt;discharging&lt;br&gt;forced mixing&lt;br&gt;impact mill</td>
</tr>
<tr>
<td>flow of fluids through granular beds</td>
<td><img src="" alt="Diagram" /></td>
<td>pneumatic silo&lt;br&gt;discharging&lt;br&gt;pneumatic homogenisation&lt;br&gt;fluidised beds</td>
</tr>
<tr>
<td>vibrations</td>
<td><img src="" alt="Diagram" /></td>
<td>vibration systems&lt;br&gt;for charging and discharging&lt;br&gt;screen machine&lt;br&gt;vibration mill</td>
</tr>
</tbody>
</table>
Mixing - of granular materials

1. drum mixer

- a) cylindrical mixing drum
- b) double cone mixer
- c) double cone mixer with a tumbling
- d) cubic mixer
- e) tetrahedral mixer
- f) V - mixer
Motion processes in a drum mixer

a) small number of revolutions  
b) higher number of revolutions  
c) number of revolutions near $n_{crit}$.
Mixing processes in a drum mixer

a) before mixing  b) after mixing  c) composition of samples taken from different heights of the mixing bulk materials, before mixing ($t = 0$), while mixing, and after mixing ($t > t_M$)
\[ s^2 = f(t) \] for different mixing processes

a) without segregation processes

b) with segregation processes in the beginning, components of higher density are above those of lower density

c) with segregation processes in the beginning, components of higher density are under those of lower density
Mixing - of granular materials

2. kneader, pan mixer

a) double-shaft kneader or pan mixer  
b) fast running paddle mixer

1 rotating mixing tool  
2 repel share, ploughshare  

A feed  
M mixed product
Mixing - of granular materials

3. Pneumatic mixer

a) fluidised bed mixer   b) air jet mixer

LE   air inlet
LA   air outlet
A    feed material for mixing
S1 – S4 separate aeration segments
Fluid Flow through Particle Beds

1. Darcy's law (development of water purification process, model: laminar permeation of groundwater through sand, Re < 0.5 ... 20):

\[ u \propto \text{grad} \, p \Rightarrow u = k \cdot \text{grad} \, p \quad \text{or} \quad \nabla \cdot \mathbf{V} = k \cdot A \cdot \text{grad} p \quad \text{(1)} \]

original Darcy (1856):

\[ \text{grad} \, p = \frac{\Delta h_w}{\Delta h_b} \quad \text{(3)} \]

\[ \dot{V} = k_f \cdot A \cdot \frac{\Delta h_w}{\Delta h_b} \quad \text{(4a)} \]

or \[ u = k_f \cdot \frac{\Delta h_w}{\Delta h_b} \quad \text{(4b)} \]

\( k_f \) – permeability

2. Permeability according to Carman and Kozeny:

\[ u = \frac{e^3}{K_{CK} \cdot \eta \cdot A_{SV}^2 \cdot (1 - e)^2} \cdot \text{grad} \, p \quad \text{(5)} \]

\[ k_f = \frac{e^3 \cdot \rho_f \cdot g}{K_{CK} \cdot \eta \cdot A_{SV}^2 \cdot (1 - e)^2} = \frac{e^3 \cdot \rho_f \cdot g \cdot d_{ST}^2}{36 K_{CK} \cdot \eta \cdot (1 - e)^2} \quad \text{(6)} \]

3. Reference values of permeability and flow behaviour (flow function \( \Phi_f \)):

<table>
<thead>
<tr>
<th>( k_f^{(1)} ) in m/s</th>
<th>permeability</th>
<th>soil behaviour</th>
<th>( \Phi_f = \frac{\sigma_f}{\sigma_c} )</th>
<th>flowability</th>
<th>( \approx d_{ST}^{(2)} ) in ( \mu m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 10^{-9}</td>
<td>practically impermeable (- 3.15 cm/a) very low (- 26 cm/mon)</td>
<td>low binding</td>
<td>0 - 2</td>
<td>very cohesive</td>
<td>0 - 0.5, 0.5 - 5</td>
</tr>
<tr>
<td>10^{-9} - 10^{-7}</td>
<td>low (- 86 cm/d)</td>
<td>low binding</td>
<td>2 - 4</td>
<td>cohesive</td>
<td>5 - 50</td>
</tr>
<tr>
<td>10^{-7} - 10^{-5}</td>
<td>medium (- 3.6 m/h) high</td>
<td>non binding</td>
<td>&gt; 4</td>
<td>easy to free flowing</td>
<td>50 - 500, 500 - 15 mm</td>
</tr>
<tr>
<td>10^{-5} - 10^{-3}</td>
<td>high</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10^{-3} - 1</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

\(^{(1)}\) according to Terzaghi / Peck
\(^{(2)}\) \( K_{CK} = 5 \) (spheres), \( \rho_f = 10^3 \text{ kg/m}^3 \), \( \eta = 10^{-3} \text{ Pa.s} \), \( e = 0.38 \)
Flow of fluids through granular beds - important mechanical micro processes

1. schematic flow of fluids through a granular bed

- $P_2 - P_1$: pressure drop across the bed
- $u$: average velocity of flow of the fluid
- $u_\varepsilon$: average velocity of flow of the fluid in the pores
- $\varepsilon$: porosity of the bed
- $A$: total cross sectional area of the bed
- $I$: thickness of the bed
2. different states of granular beds while fluid permeation

- bulk material
- begin of fluidisation
- homogeneous fluidised bed
- inhomogeneous fluidised bed
- unsteady fluidised bed

- gas or liquid
- liquid
- gas
- gas or liquid

- low velocity of flow of the fluid
- high velocity of flow of the fluid
3. pressure drop in the granular bed dependent on the fluid flow rate
# Fluid Flow through Particle Beds – Permeation Models

## a) Fixed particle bed

<table>
<thead>
<tr>
<th>No</th>
<th>Author</th>
<th>Equation</th>
<th>Valid for</th>
<th>Model of</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Darcy</td>
<td>$\frac{\Delta p}{h_B} = k \cdot u = K \cdot \eta \cdot u, \quad K = \text{Darcy constant}$</td>
<td>$+$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>Carman, Kozeny</td>
<td>$\frac{\Delta p}{h_B} = \left(1 - \varepsilon\right)^2 \cdot \frac{A_{s,v}^2 \cdot \eta \cdot u \cdot K_{\text{CK}}}{\varepsilon^3}, \quad K_{\text{CK}} = \text{Carman Kozeny constant, } K_{\text{CK}} = 5 \text{ for spheres of equal size with small deviation}$</td>
<td>$+$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>Gupte</td>
<td>$\frac{\Delta p}{\rho_f \cdot u^2} = \frac{5.6}{h_B \cdot \varepsilon^{5.5}}$</td>
<td>$+$</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>4</td>
<td>Molerus, Pahl, Rumpf</td>
<td>$\frac{\Delta p}{\rho_f \cdot u^2} = 4 \cdot (1 - \varepsilon) \cdot \frac{5.6}{h_B \cdot \varepsilon^{4.55}}$</td>
<td>$+$</td>
<td>-</td>
<td>0.35 - 0.7</td>
</tr>
<tr>
<td>Experiments with fine disperse particle beds</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------------------------------------------</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Pärnt</td>
<td>$\frac{\Delta p}{h_B} = \xi \cdot Re_h \cdot \frac{\eta \cdot u}{\psi_F^2 \cdot \psi_R^2 \cdot d_{ST}^2 / \varepsilon^3}$, $Re_h = \frac{Re}{1 - \varepsilon}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Burke, Plummer</td>
<td>$\frac{\Delta p}{\rho_f \cdot u^2} \cdot \frac{d_{K}}{h_B} = 1.75 \cdot \frac{1 - \varepsilon}{\varepsilon^3}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ergun</td>
<td>$\frac{\Delta p}{\rho_f \cdot u^2} \cdot \frac{d_{K}}{h_B} = \lambda \cdot \frac{(1 - \varepsilon)}{\varepsilon^3}$, $\lambda = 150 \cdot \frac{1 - \varepsilon}{Re} + 1.75$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Molerus</td>
<td>Euler number of fixed bed: $Eu_B = \frac{24}{Re} \left{1 + 0.692 \cdot \left[\frac{d}{a} + \frac{1}{2} \cdot \left(\frac{d}{a}\right)^2\right]<em>{0.95}\right} + \frac{4}{\sqrt{Re}} \left{1 + 0.12 \cdot \left(\frac{d}{a}\right)^{1.5}</em>{0.95}\right} + 0.4 \left(\frac{d}{a}\right)_{0.95} \cdot \frac{0.891 \cdot \varepsilon}{Re^{0.1}}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Re &lt; 10$^4$</td>
<td>$Re = \frac{(u / \varepsilon) \cdot d \cdot \rho_f}{\eta}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surface separation ratio</td>
<td>$\left(\frac{d}{a}\right)_{0.95} = \sqrt[0.95]{\frac{1 - \varepsilon}{1 - \varepsilon}}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**b) Fluidized bed**

<table>
<thead>
<tr>
<th>No</th>
<th>Author</th>
<th>Equation</th>
<th>Valid for</th>
<th>Model of</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>Beranek, Rose, Winterstein</td>
<td>$\Delta p = \frac{(1 - \varepsilon_L) \cdot (\rho_s - \rho_f) \cdot g}{h_B}$</td>
<td>$\mathrm{Re} &lt; 0.5$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\varepsilon_L = \text{Porosity at point of bed expansion}$</td>
<td>$0.5 - 1000$</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\varepsilon_L = \text{Porosity at point of bed expansion}$</td>
<td>$\mathrm{Re} &gt; 1000$</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>10</td>
<td>Molerus</td>
<td>$Eu_w = \frac{24 \cdot \rho_s - \rho_f}{\mathrm{Re} \cdot \left[ \frac{d}{a} + \frac{1}{2} \left( \frac{d}{a} \right)^2 \right]} + \frac{4}{\sqrt{\mathrm{Re}}} \cdot \left[ 1 + 0.07 \cdot \left( \frac{d}{a} \right)^{1.5} \right] + 0.4 + \frac{d}{a} \cdot 0.907 \cdot \mathrm{Re}^{0.1}$</td>
<td>$\mathrm{Re} &lt; 10^4$</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Euler number of fluidized bed: $Eu_w = \frac{4 \cdot \rho_s - \rho_f}{3 \cdot \rho_f} \cdot \frac{d \cdot g}{(u/\varepsilon)^2}$ with $\lim_{\varepsilon \to 1} Eu_w = c_w$</td>
<td>$0.5 - 1$</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

**Remarks**
- $\eta = \frac{(u/\varepsilon) \cdot \rho_f}{\rho_f}$
- Surface separation ratio $\left( \frac{d}{a} \right)_{\eta} = \frac{\sqrt{1 - \varepsilon}}{0.9 - \sqrt{1 - \varepsilon}}$
**Fluidized Bed Behaviours of Powders and Particulate Solids according to Geldart**

---

**Group properties**

**A**  
- Powders of small particle size and/or low density (d = 20 - 200 \(\mu\)m, \(\rho_s < 2000\) kg/m\(^3\)).
- Formation of bubbles only after remarkable expansion of the fluidised bed and above a minimum fluidisation velocity, formation of small bubbles.
- After gas feed switch off slow collapsing of fluidised bed (deaeration rate about 3 - 6 mm/s).

**B**  
- Particulate solids of medium particle size and density (d = 40 - 500 \(\mu\)m, \(\rho_s < 1400 - 4000\) kg/m\(^3\)).
- Formation of bubbles just above a minimum fluidisation velocity.
- Bubble velocity faster than gas velocity in voids between particles.
- After gas feed switch off fast collapsing of fluidised bed.

**C**  
- Cohesive powders and particulate solids.
- Very bad fluidisation behaviour because of dominant adhesion forces.
- Inhomogeneous gas permeation through the bed by formation of gas channels.

**D**  
- Particulate solids of coarse particle size and/or large density (d > 400 \(\mu\)m, \(\rho_s > 4000\) kg/m\(^3\)).
- High gas velocity necessary for fluidisation.
- Bubble velocity slower than gas velocity in the voids between particles.

---

Molerus, O.: Fluid-Feststoff-Strömungen, Springer-Verlag, Heidelberg 1982
Survey of constitutive functions, processing and handling problems of cohesive powders.

<table>
<thead>
<tr>
<th>Property, problems</th>
<th>Physical principle</th>
<th>Physical law</th>
<th>Physical assessment of product quality</th>
<th>Value range</th>
<th>Evaluation</th>
<th>Particle size d in μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large adhesion potential</td>
<td>$F_{H0} = \frac{C_{H,S,SLS}}{2\pi g \cdot a_d \cdot \rho_s \cdot d^2}$</td>
<td>Adhesion $(100 \text{μm})^2$</td>
<td>Weight $d^2$</td>
<td>1 - 100</td>
<td>slightly adhesive</td>
<td>10 - 100</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>100 - 10$^4$</td>
<td>adhesive</td>
<td>1 - 10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10$^4$ - 10$^8$</td>
<td>very adhesive</td>
<td>0.01 - 1</td>
</tr>
<tr>
<td>Large intensification of adhesion</td>
<td>$\kappa = \frac{\kappa_p}{\kappa_A - \kappa_p}$</td>
<td>Contact consolidation coefficient $\kappa$ by flattening</td>
<td></td>
<td>0.1 – 0.3</td>
<td>soft</td>
<td>&lt; 10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.3 – 0.77</td>
<td>very soft</td>
<td>&lt; 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>&gt; 0.77</td>
<td>extreme soft</td>
<td>&lt; 0.1</td>
</tr>
<tr>
<td>Poor flowability</td>
<td>$\sigma = \frac{\sigma_1}{\sigma_c}$</td>
<td>Flow function $f_f$</td>
<td></td>
<td>2 - 4</td>
<td>cohesive</td>
<td>&lt; 100</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1 - 2</td>
<td>very cohesive</td>
<td>&lt; 10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>&lt; 1</td>
<td>non-flowing</td>
<td>&lt; 0.1</td>
</tr>
<tr>
<td>Large compressibility</td>
<td>$\rho = \left(1 + \frac{\sigma_{M,ST}}{\sigma_0}\right)^n$</td>
<td>Compressibility index $n$</td>
<td></td>
<td>0.05 – 0.1</td>
<td>compressible</td>
<td>&lt; 100</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.1 - 1</td>
<td>very compressible</td>
<td>&lt; 10</td>
</tr>
<tr>
<td>Small permeability</td>
<td>$u = k_f \cdot \frac{\Delta h}{\Delta h_b}$</td>
<td>Permeability $k_f$ in m/s</td>
<td></td>
<td>$&lt; 10^{-9}$</td>
<td>non-permeable</td>
<td>&lt; 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$10^{-9}$ - $10^{-7}$</td>
<td>very low</td>
<td>1 - 10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$10^{-7}$ - $10^{-5}$</td>
<td>low</td>
<td>10 - 100</td>
</tr>
<tr>
<td>Poor fluidisation</td>
<td>$\Delta p = f(u(d_p))$</td>
<td>Channelling</td>
<td></td>
<td></td>
<td>Group C,</td>
<td></td>
</tr>
</tbody>
</table>