4. Particle separation in a fluid flow

4.1 Single particle flow in a fluid and flow-around pattern

4.1.1 Stationary particle sedimentation

4.1.2 Uniformly accelerated particle sedimentation

4.1.3 Swarm confinement of particle clusters

4.1.4 Fluid flow through particle beds

4.2 Micro- and macroturbulence

4.3 Particle diffusion in a dispersion medium (fluid)

4.4 Dynamics of particle transport in turbulent fluids (turbulent particle dispersion or diffusion)

4.5 Particle separation efficiency in a turbulent flow field of an apparatus

4.5.1 Separation function of turbulent cross flow separation

4.5.2 Separation function of counter-current separation

4.6 Machines and apparatuses of particle classification
Particle Flow in a Fluid

Co-current:
- Field force $\vec{F}_F$
- Fluid stream
- Coordinate $y$
- Particle flow $\vec{v}$
- Relative "co-current particle flow"

Pending:
- Field force $\vec{F}_F$
- Fluid flow $\vec{u}$

Counter-current:
- Field force $\vec{F}_F$
- Fluid flow $\vec{u}$
- Particle absolute velocity $\vec{v}_a = \vec{u} - \vec{v}$

Cross flow:
- Field force $\vec{F}_F$
- Relative onflow velocity of fluid $\vec{u}_r = \vec{u} - \vec{v}$

Acting forces:
- Dynamic buoyancy $\vec{F}_D$
- Static buoyancy $\vec{F}_A$
- Fluid drag $\vec{F}_W$
- Resulting force $\vec{F}_R$
Flow-around of Smooth Spheres

BERNOULLI-Equation: \( p_{\text{stat}} + \rho_f u^2/2 + \rho_f g y = \text{const.} \)

1. Effect of dynamic buoyancy \( F_D = c_D \cdot A_p \cdot \rho_f u^2/2 \)

a) Particle rotation

b) Non-uniform onflow of symmetric sphere

c) Non-uniform onflow profile of asymmetric body
Flow-around Pattern of Smooth Spheres

2. Flow-around ranges
Prerequisites: uniform, laminar and stationary onflow of smooth sphere at rest

I) Viscous flow-around pattern, Re < 0.25, STOKES

\[ \text{Re} = \frac{u \cdot d \cdot \rho_f}{\eta} \]

\[ c_w = \frac{24}{\text{Re}} \]

\[ F_w = c_w \cdot A_p \cdot \rho_f \cdot u^2 / 2 \]

II) Transition regime, 0.25 < Re < 10^3
IIa) Laminar flowing eddies, 24 < Re < 130

IIb) Eddy separation (separation point A), instationary eddy shedding, vortex street

130 < Re < 1000

III) Square range of inertia, 10^3 < Re < 2 \cdot 10^5, NEWTON

\[ c_w = 0.44 \]

for Re < 2 \cdot 10^5

\[ c_w = \frac{24}{\text{Re}} + \frac{4}{\sqrt{\text{Re}}} + 0.4 \]

or

\[ c_w = \frac{24}{\text{Re}} + \frac{32}{\sqrt{\text{Re}}} + \frac{1}{3} \]

IV) Range of turbulent boundary layer flow at onflow (transition point U):

2 \cdot 10^5 < Re < 4 \cdot 10^5

\[ c_w = 0.07 \text{ to } 0.3 \]
To 4.1.1: Equivalent Falling Classes of Particles

- balance of the forces of buoyancy, weight and fluid resistance
- correlation between particle size \(d\) and the quasi-stationary settling velocity \(v_s\) in the field of gravity \(g\):

\[
v_s^2 = \frac{2 \cdot (\rho_s - \rho_f) \cdot V_p}{c_w \cdot \rho_f \cdot A_p} \cdot g
\]

\(A_p\) onflow cross-sectional area of the particle
\(c_w\) drag coefficient of the fluid flow pattern around the particle
\(V_p\) particle volume
\(\rho_f, \rho_s\) fluid and solid density

- for constant particle shape, “large” (i+1) and “lightweight” (L) particles settle just as fast as “small” (i) and “heavy” (S) particles.

\[
v_s(d_{i+1}, \rho_{s,L}) = v_s(d_i, \rho_{s,S})
\]

- Depending on the particle flow-around patterns \(v_s \propto d^\alpha\) and with

\[
c_w \propto Re^{\frac{1-2\alpha}{3}}
\]

\[
Re = v_s \cdot d \cdot \rho_f / \eta_f \quad \text{particle Reynolds number}
\]

Equivalent-falling condition dependent on the particle flow-around pattern

<table>
<thead>
<tr>
<th>Exponent (\alpha)</th>
<th>(\frac{\alpha + 1}{3 \cdot \alpha})</th>
<th>Flow pattern</th>
<th>Reynolds number</th>
<th>Drag coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1/2</td>
<td>laminar (Stokes)</td>
<td>(Re &lt; 1)</td>
<td>(c_w \propto Re^{-1})</td>
</tr>
<tr>
<td>(1/2 &lt; \alpha &lt; 2)</td>
<td>1/2 ... 1</td>
<td>transition</td>
<td>(1 &lt; Re &lt; 10^3)</td>
<td>(c_w \propto Re^{-1...0})</td>
</tr>
<tr>
<td>1/2</td>
<td>1</td>
<td>turbulent (Newton)</td>
<td>(10^3 &lt; Re &lt; (2 - 4) \cdot 10^5)</td>
<td>(c_w \propto Re^0)</td>
</tr>
</tbody>
</table>

\[
\frac{d_{i+1}}{d_i} = \left(\frac{\rho_{s,L} - \rho_f}{\rho_s - \rho_f}\right)^{\frac{\alpha + 1}{3 \alpha}} \quad \text{or} \quad \frac{v_s}{v_{sT}} = \left(\frac{d_T}{d}\right)^\alpha = \left(\frac{\rho_s - \rho_f}{\rho_{s,T} - \rho_f}\right)^{\frac{\alpha + 1}{3}}
\]
3. Drag coefficient $c_w = f(Re)$

4. Influence of turbulence intensity of particle onflow on the drag coefficient $c_w$ of moving spheres or spheres at rest
Flow-around of Single Particles

5. Ljascenko number \( L_j = \Omega = f(Ar) \) of smooth spheres

\[
L_j = 4 \cdot \frac{\rho_f}{\rho_s - \rho_f} \cdot \frac{c_w}{\nu^2}
\]

\[
\nu = \frac{\eta}{\rho_f}
\]

\[
Ar = \frac{3}{4} \cdot \frac{Re^2 \cdot c_w}{\nu^2} \cdot \frac{d^3 \cdot g \cdot \rho_s - \rho_f}{\rho_f}
\]

6. Particle shape coefficient \( k_\psi \) of stationary settling velocity \( v_{s,\psi} = k_\psi \cdot v_{s, \text{sphere}} \)

<table>
<thead>
<tr>
<th>Body shape</th>
<th>Equivalent sphere diameter ( d_v )</th>
<th>Shape factor ( \psi_\Lambda )</th>
<th>Shape coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>d</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Cube</td>
<td>1,241 a</td>
<td>0,806</td>
<td>0,92</td>
</tr>
<tr>
<td>Parallel epiped</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a x a x 2 a</td>
<td>1,563 a</td>
<td>0,767</td>
<td>0,90</td>
</tr>
<tr>
<td>a x 2 a x 2 a</td>
<td>1,970 a</td>
<td>0,761</td>
<td>0,89</td>
</tr>
<tr>
<td>a x 2 a x 3 a</td>
<td>2,253 a</td>
<td>0,725</td>
<td>0,88</td>
</tr>
<tr>
<td>a x a x 0,1 a</td>
<td>0,576 a</td>
<td>0,435</td>
<td>0,70</td>
</tr>
<tr>
<td>a x a x 0,01 a</td>
<td>0,267 a</td>
<td>0,110</td>
<td>0,19</td>
</tr>
<tr>
<td>Cylinder</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h = 2 d</td>
<td>1,442 d</td>
<td>0,831</td>
<td>0,93</td>
</tr>
<tr>
<td>h = 1 d</td>
<td>1,145 d</td>
<td>0,875</td>
<td>0,95</td>
</tr>
<tr>
<td>h = 0,5 d</td>
<td>0,909 d</td>
<td>0,826</td>
<td>0,93</td>
</tr>
<tr>
<td>h = 0,15 d</td>
<td>0,608 d</td>
<td>0,570</td>
<td>0,79</td>
</tr>
<tr>
<td>h = 0,01 d</td>
<td>0,247 d</td>
<td>0,120</td>
<td>0,22</td>
</tr>
</tbody>
</table>
### Dimensionless Groups and their Significance

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Formula</th>
<th>Physical interpretation</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Archimedes number</td>
<td>Ar</td>
<td>( g \cdot d^3 (\rho_p - \rho_f) \cdot \rho_f / \eta^2 )</td>
<td>inertial force · buoyancy force (viscous force)^2</td>
<td>Particle settling</td>
</tr>
<tr>
<td>Bingham number</td>
<td>Bm</td>
<td>( \tau_0 \cdot d / \eta \cdot u )</td>
<td>yield stress (viscous force)</td>
<td>Flow of Bingham fluids = yield number</td>
</tr>
<tr>
<td>Bingham Reynolds number</td>
<td>Re_B</td>
<td>( d \cdot u \cdot \rho_f / \eta )</td>
<td>inertial force (viscous force)</td>
<td>Flow of Bingham fluids (i.e. viscoplastics)</td>
</tr>
<tr>
<td>Blake number</td>
<td>B</td>
<td>( u \cdot \rho_f / \eta \cdot (1 - \varepsilon) \cdot d )</td>
<td>inertial force (viscous force)</td>
<td>Flow through particle beds</td>
</tr>
<tr>
<td>Bond number</td>
<td>B_d</td>
<td>( \left( \rho_f - \rho_g \right) \cdot d^2 \cdot g / \sigma_{lg} )</td>
<td>gravitational force (surface tension force)</td>
<td>Atomization = Eotvos number, Eo</td>
</tr>
<tr>
<td>Capillary number</td>
<td>Ca</td>
<td>( \eta \cdot u / \sigma_{lg} )</td>
<td>viscous force (surface tension force)</td>
<td>Two-phase flow, free surface flow</td>
</tr>
<tr>
<td>Cauchy number</td>
<td>C</td>
<td>( \rho_f \cdot u^2 / \beta )</td>
<td>inertial force (compressibility force)</td>
<td>Compressible flow, hydraulic transients</td>
</tr>
<tr>
<td>Cavitation number</td>
<td>( \sigma )</td>
<td>( \frac{p - p_e}{p_f \cdot u^2 / 2} )</td>
<td>excess pressure above vapor pressure velocity head</td>
<td>Cavitation</td>
</tr>
<tr>
<td>Centrifuge number</td>
<td>( z )</td>
<td>( \frac{R \cdot \omega^2}{g} )</td>
<td>centrifugal force (gravity force)</td>
<td>Centrifugal fields, = Froude number</td>
</tr>
<tr>
<td>Dean number</td>
<td>( D_c )</td>
<td>( \frac{Re}{(D_c / D_R)^{1/2}} )</td>
<td>Reynolds number - centrifugal force (inertial force)</td>
<td>Flow in curved channels</td>
</tr>
<tr>
<td>Deborah number</td>
<td>( D_e )</td>
<td>( t_{\text{rel}} \cdot \omega )</td>
<td>fluid relaxation time (flow characteristic time)</td>
<td>Viscoelastic flow</td>
</tr>
<tr>
<td>Degree of turbulence</td>
<td>( \text{Tu} )</td>
<td>( \sqrt{\frac{u^2}{\nu}} \cdot \frac{u}{\nu} )</td>
<td>root mean squared of flow rate fluctuations (fluid flow rate)</td>
<td>Turbulence intensity</td>
</tr>
<tr>
<td>Drag coefficient</td>
<td>( c_w )</td>
<td>( \frac{F_w}{A_p \cdot \rho_f \cdot u^2 / 2} )</td>
<td>fluid drag force (projected area · velocity head)</td>
<td>Flow-around objects, particle settling</td>
</tr>
<tr>
<td>Elasticity number</td>
<td>( \text{El} )</td>
<td>( t_{\text{rel}} \cdot \eta / \rho_f \cdot u^2 )</td>
<td>elastic force (inertial force)</td>
<td>Viscoelastic flow</td>
</tr>
<tr>
<td>Euler number</td>
<td>( \text{Eu} )</td>
<td>( \frac{\Delta p}{\rho_f \cdot u^2} )</td>
<td>frictional pressure loss (2 · velocity head)</td>
<td>Fluid friction in conduits</td>
</tr>
<tr>
<td>Fanning friction factor</td>
<td>( f )</td>
<td>( \frac{D_f \cdot \Delta p}{2 \cdot \rho_f \cdot u^2 \cdot d} = \frac{2 \cdot \tau_w}{\rho_f \cdot u^2} )</td>
<td>wall shear stress (velocity head)</td>
<td>Fluid friction in conduits, Darcy friction factor = 4f</td>
</tr>
<tr>
<td>Froude number</td>
<td>( \text{Fr} )</td>
<td>( \frac{u^2}{g \cdot R} )</td>
<td>inertial force (gravity force)</td>
<td>Often defined as ( \text{Fr} = \frac{u}{\sqrt{g \cdot R}} )</td>
</tr>
<tr>
<td>Densometric Froude number</td>
<td>( \text{Fr'} )</td>
<td>( \frac{\rho_f \cdot u^2}{(\rho_p - \rho_f) \cdot g \cdot d} )</td>
<td>inertial force (gravity force)</td>
<td>( \text{Fr'} = \frac{u}{\sqrt{(\rho_p - \rho_f) \cdot g \cdot d / \rho_f}} )</td>
</tr>
<tr>
<td>Hedström number</td>
<td>He</td>
<td>( \frac{d^2 \cdot \tau_0 \cdot \rho_f}{\eta^2} )</td>
<td>Bingham Reynolds number</td>
<td>Bingham number</td>
</tr>
<tr>
<td>----------------</td>
<td>----</td>
<td>-----------------------------------------------</td>
<td>------------------------</td>
<td>----------------</td>
</tr>
<tr>
<td>Hodgson number</td>
<td>H</td>
<td>( \frac{V \cdot \omega \cdot \Delta p}{V \cdot \pi} )</td>
<td>time constant of system period of pulsation</td>
<td>( \frac{(\text{inertial force})^2}{\text{viscous force} \cdot \text{fluid drag force}} )</td>
</tr>
<tr>
<td>Ljáščenko Number</td>
<td>Lj</td>
<td>( \frac{\nu^2}{\eta} \cdot \frac{\rho_f^2}{\rho_p \cdot \rho_f} )</td>
<td>( \frac{\rho_f}{\eta} \cdot \frac{\rho_p \cdot \rho_f}{\nu} )</td>
<td>Particle settling, ( 4 \cdot \frac{\text{Re}}{3 \cdot c_w} )</td>
</tr>
<tr>
<td>Mach number</td>
<td>M</td>
<td>( \frac{u}{c_s} )</td>
<td>fluid velocity sonic velocity</td>
<td>Flow of compressible fluids</td>
</tr>
<tr>
<td>Newton number</td>
<td>Ne</td>
<td>( \frac{F_w}{\rho_f \cdot A_p \cdot u^2} )</td>
<td>fluid drag force inertial force</td>
<td>Flow-around of particles, ( c_w ) fluid drag coefficient</td>
</tr>
<tr>
<td>Ohnesorge number</td>
<td>Z</td>
<td>( \frac{\eta}{(\rho_f \cdot d \cdot \sigma_{ig})^{1/2}} )</td>
<td>viscous force ( (\text{inertial force} \cdot \text{surface tension force})^{1/2} )</td>
<td>Atomization = Weber number Reynolds number</td>
</tr>
<tr>
<td>Particle Peclet number</td>
<td>PeP</td>
<td>( \frac{\nu_s \cdot d}{D_p} ), ( D_p = \frac{k_b T}{3 \pi \eta \cdot d} )</td>
<td>convective transport diffusive transport</td>
<td>Heat, mass transfer, mixing, = Bodenstein number Bo</td>
</tr>
<tr>
<td>Pipeline parameter</td>
<td>Pn</td>
<td>( \frac{v \cdot u_o}{2 \cdot g \cdot H} )</td>
<td>maximum water – hammer pressure rise 2 static pressure</td>
<td>Water “hammer”</td>
</tr>
<tr>
<td>Power number</td>
<td>( c_p )</td>
<td>( \frac{P}{\rho_f \cdot n^3 \cdot D_A^3} )</td>
<td>impeller drag force inertial force</td>
<td>Agitation</td>
</tr>
<tr>
<td>Prandtl velocity ratio</td>
<td>( u^+ )</td>
<td>( \frac{u}{(\tau_w / \rho_f)^{1/2}} )</td>
<td>velocity normalized by friction velocity</td>
<td>Turbulent flow near a wall, friction velocity = ( \sqrt{\tau_w / \rho_f} )</td>
</tr>
<tr>
<td>Reynolds number</td>
<td>Re</td>
<td>( \frac{d \cdot u \cdot \rho_f}{\eta} )</td>
<td>inertial force viscous force</td>
<td>Fluid flow</td>
</tr>
<tr>
<td>Schmidt number</td>
<td>Sc</td>
<td>( \frac{D_t}{\nu} )</td>
<td>diffusive transport viscous friction</td>
<td>Turbulent Schmidt number</td>
</tr>
<tr>
<td>Stokes number</td>
<td>St</td>
<td>( Cu \cdot \frac{d^2 \cdot u \cdot \rho_s}{18 \cdot \eta \cdot D} )</td>
<td>particle inertial force fluid drag force</td>
<td>Particle impact in fluid flow against tool</td>
</tr>
<tr>
<td>Strouhal number</td>
<td>St</td>
<td>( \frac{f \cdot D_g}{u} )</td>
<td>vortex shedding frequency characteristic flow time scale</td>
<td>Vortex shedding, von Karman vortex streets</td>
</tr>
<tr>
<td>Weber number</td>
<td>We</td>
<td>( \frac{\rho_f \cdot u^2 \cdot d}{\sigma_{ig}} )</td>
<td>inertial force surface tension force</td>
<td>Bubble, drop formation</td>
</tr>
</tbody>
</table>

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>SI Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Acceleration</td>
<td>m/s²</td>
</tr>
<tr>
<td>A_p</td>
<td>Projected particle area</td>
<td>m</td>
</tr>
<tr>
<td>c_s</td>
<td>Sonic velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>d</td>
<td>Characteristic particle dimension (diameter)</td>
<td>μm</td>
</tr>
<tr>
<td>D_A</td>
<td>Diameter of agitator</td>
<td>m</td>
</tr>
</tbody>
</table>

Fig. 4.9
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_B$</td>
<td>Characteristic width of flow channel</td>
<td>m</td>
</tr>
<tr>
<td>$D_R$</td>
<td>Diameter of pipe or process chamber</td>
<td>m</td>
</tr>
<tr>
<td>$D_c$</td>
<td>Diameter of flow channel curvature</td>
<td>m</td>
</tr>
<tr>
<td>$D$</td>
<td>Diffusivity</td>
<td>m²/s</td>
</tr>
<tr>
<td>$D_t$</td>
<td>Turbulent Diffusion coefficient</td>
<td>m²/s</td>
</tr>
<tr>
<td>$f'$</td>
<td>Vortex shedding frequency</td>
<td>1/s</td>
</tr>
<tr>
<td>$F_W$</td>
<td>Drag force</td>
<td>N</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration of gravity</td>
<td>m/s</td>
</tr>
<tr>
<td>$H$</td>
<td>Static head (height of isostatic pressure)</td>
<td>m</td>
</tr>
<tr>
<td>$n$</td>
<td>Rotational speed or number of revolutions</td>
<td>1/s</td>
</tr>
<tr>
<td>$p$</td>
<td>Pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$p_v$</td>
<td>Vapor pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>Average static pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$\Delta p$</td>
<td>Frictional pressure drop</td>
<td>Pa</td>
</tr>
<tr>
<td>$P$</td>
<td>Power</td>
<td>W</td>
</tr>
<tr>
<td>$R$</td>
<td>Radius of process chamber or apparatus</td>
<td>m</td>
</tr>
<tr>
<td>$t_{relax}$</td>
<td>Fluid relaxation time</td>
<td>s</td>
</tr>
<tr>
<td>$u$</td>
<td>Local fluid velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$\bar{u}$</td>
<td>Characteristic or average fluid velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$v$</td>
<td>Wave propagation speed</td>
<td>m/s</td>
</tr>
<tr>
<td>$v_s$</td>
<td>Particle settling velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$V$</td>
<td>Volume of process chamber</td>
<td>m³</td>
</tr>
<tr>
<td>$V_p$</td>
<td>Particle volume</td>
<td>m³</td>
</tr>
<tr>
<td>$\bar{V}$</td>
<td>Average volumetric flow rate</td>
<td>m³/s</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Bulk compression modulus</td>
<td>Pa</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Porosity, void fraction</td>
<td>m³/ m³</td>
</tr>
<tr>
<td>$\eta$</td>
<td>(dynamic) fluid viscosity</td>
<td>Pa · s</td>
</tr>
<tr>
<td>$\eta_p$</td>
<td>Infinite shear viscosity (Bingham fluid, $\dot{\gamma} \to \infty$)</td>
<td>Pa · s</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic fluid viscosity</td>
<td>m²/s</td>
</tr>
<tr>
<td>$\rho_f$</td>
<td>Fluid density</td>
<td>kg/m³</td>
</tr>
<tr>
<td>$\rho_g$, $\rho_l$</td>
<td>Gas, liquid densities</td>
<td>kg/m³</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>Particle or dispersed phase density</td>
<td>kg/m³</td>
</tr>
<tr>
<td>$\sigma_{lg}$</td>
<td>Surface tension</td>
<td>N/m</td>
</tr>
<tr>
<td>$\tau_0$</td>
<td>Yield shear stress of Bingham fluid</td>
<td>Pa</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Characteristic frequency or reciprocal time scale of flow</td>
<td>1/s</td>
</tr>
</tbody>
</table>

### 4.1.2 Survey about Models of Uniformly Accelerated Particle Sedimentation (TOMAS 2010)

<table>
<thead>
<tr>
<th>Microprocess variables</th>
<th>Laminar Flow-around of Particles</th>
<th>Turbulent Flow-around of Particles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reynolds number, $c_W$</td>
<td>$Re_{St} &lt; 0.25 - 1$, $c_W = 24/Re_{St}$</td>
<td>$10^3 &lt; Re_N &lt; Re_c = 2 \cdot 10^5$, $c_W = 0.44$</td>
</tr>
<tr>
<td>Stationary Settling Velocity</td>
<td>$v_{s,St} = \frac{(\rho_s - \rho_f) \cdot d^2 \cdot g}{18 \eta}$</td>
<td>$v_{s,N} = \frac{4 \cdot (\rho_s - \rho_f) \cdot d \cdot g}{3 \cdot c_W \cdot \rho_f}$</td>
</tr>
<tr>
<td>Particle Size Range</td>
<td>$d_{St} \leq \frac{18 \cdot \eta^2 \cdot Re_{St}}{\rho_f \cdot (\rho_s - \rho_f)}$</td>
<td>$d_N \geq \frac{3 \cdot c_W \cdot \eta^2 \cdot Re_N^2}{4 \cdot \rho_f \cdot (\rho_s - \rho_f)}$</td>
</tr>
<tr>
<td>Differential Equation</td>
<td>$\frac{dv(t)}{dt} = g \cdot \left(1 - \frac{v}{v_s}\right)$</td>
<td>$\frac{dv(t)}{dt} = g \cdot \left(1 - \frac{v^2}{v_s^2}\right)$</td>
</tr>
<tr>
<td>Velocity-Time Law</td>
<td>$v(t) = v_s \cdot \left[1 - \exp\left(-\frac{t}{t_{63,v_s}}\right)\right]$</td>
<td>$v(t) = v_s \cdot \tanh\left(\frac{t}{t_{76,v_s}}\right)$</td>
</tr>
<tr>
<td>Characteristic Settling Time</td>
<td>$t_{63,v_s} = \frac{v_s}{g} \cdot \frac{(\rho_s - \rho_f) \cdot d^2}{18 \eta}$</td>
<td>$t_{76,v_s} = \frac{v_s}{g} \cdot \frac{4 \cdot (\rho_s - \rho_f) \cdot d}{3 \cdot c_W \cdot \rho_f \cdot g}$</td>
</tr>
<tr>
<td>Characteristic Settling Velocities</td>
<td>$v(t = t_{63,v_s}) = v_s \cdot [1 - \exp(-1)] = 0.63 \cdot v_s$</td>
<td>$v(t = t_{76}) = v_s \cdot \tanh(1) = 0.76 \cdot v_s$</td>
</tr>
<tr>
<td></td>
<td>$v(t_{95}) = 3 \cdot t_{63,v_s} = v_s \cdot [1 - \exp(-3)] = 0.95 \cdot v_s$</td>
<td>$v(t_{96} = 2 \cdot t_{76,v_s}) = v_s \cdot \tanh(2) = 0.964 \cdot v_s$</td>
</tr>
<tr>
<td>Differential Equation</td>
<td>$\frac{ds(t)}{dt} = v_s \cdot \left[1 - \exp\left(-\frac{t}{t_{63,v_s}}\right)\right]$</td>
<td>$\frac{ds(t)}{dt} = v_s \cdot \tanh\left(\frac{t}{t_{76,v_s}}\right)$</td>
</tr>
<tr>
<td>Distance-Time Law</td>
<td>$s(t) = v_s \cdot \left[t - t_{63,v_s} \cdot \left[1 - \exp\left(-\frac{t}{t_{63,v_s}}\right)\right]\right]$</td>
<td>$s(t) = v_s \cdot t_{76,v_s} \cdot \ln\cosh\left(\frac{t}{t_{76,v_s}}\right)$</td>
</tr>
<tr>
<td>Characteristic Acceleration Distances</td>
<td>$s(t = t_{63,v_s}) = 0.37 \cdot v_s \cdot t_{63,v_s} = 0.37 \cdot v_s^2 / g$</td>
<td>$s(t_{76}) = 0.433 \cdot v_s \cdot t_{76,v_s} = 0.433 \cdot v_s^2 / g$</td>
</tr>
<tr>
<td></td>
<td>$s(t_{95}) = 3 \cdot t_{63,v_s} = 2.05 \cdot v_s \cdot t_{63,v_s} = 2.05 \cdot v_s^2 / g$</td>
<td>$s(t_{96}) = 1.33 \cdot v_s \cdot t_{76,v_s} = 1.33 \cdot v_s^2 / g$</td>
</tr>
</tbody>
</table>
7. Swarm confinement at sedimentation of particle cluster

a) Free flow-around of particle swarm

- Free flow-around of particle swarm
- Fluid drag coefficient $c_w$ decreases
- Settling velocity $v_s$ increases

b) Confined flow field, permeation of particle swarm

- Confined flow field, permeation of particle swarm
- Fluid drag coefficient $c_w = f(\phi_s)$
- Settling velocity $v_s = f(\phi_s)$

8. Zone sedimentation of particle bed

Sedimentation and permeation (flow-through) of comparatively dense, agglomerated particle layers

- Fluid drag force $F_w \rightarrow \Delta p$ pressure drop
- Fluid drag coefficient $c_w \rightarrow \text{Euler number } \frac{\Delta p}{(\rho_f u^2)} = f(\phi_s, d_{pore})$
- Settling velocity $v_s = f(\phi_s) \neq f(d)$

\[ \text{Height } h(t) \quad h(t) = h_0 - s(t) \]

- Accelerated
- Stationary: $v_s = \frac{dh}{dt} = \text{const.}$
- Decelerated
- Thickened sludge

$h_{DS}(t)$
9. Ratio of settling velocities of smooth spheres

\[ \frac{v_{se}}{v_s} = k_G \cdot k_T \]

a) as function of particle volume fraction \( \varphi_s \)
in a monodisperse suspension (\( k_G \) counter-current factor, \( k_T \) swarm turbulence factor)
(acc. to Brauer and Thiele)

b) as function of particle volume fraction \( \varphi_s \)
for 2 size fractions with \( \varphi_{s,G} = \varphi_{s,F} \) and \( \frac{d_G}{d_F} \) as parameter
(acc. to Brauer and Thiele)

10. Comparison of various formulas to include swarm confinement in monodisperse suspensions
Force Balance of Particle Sedimentation in a static Fluid at **uniform** (stationary)

**Onflow** and (Statistically) Homogeneous **Flow-Around** and **Flow-Through**

<table>
<thead>
<tr>
<th>Forces</th>
<th>Microscopic Particle Flow-around</th>
<th>Macroscopic Particle Bed Flow-through</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle model</td>
<td>Smooth sphere</td>
<td>Statistically homogeneous particle bed</td>
</tr>
<tr>
<td>Sink model</td>
<td>Single particle sedimentation</td>
<td>Zone sedimentation</td>
</tr>
</tbody>
</table>

### Gravity

\[
F_G = \rho_s \cdot V_p \cdot g
\]

\[
\frac{F_G}{A} = \rho_{tr} \cdot g \cdot dy
\]

### Buoyancy

\[
F_A = \rho_f \cdot V_p \cdot g
\]

\[
\frac{F_A}{A} = \rho_f \cdot g \cdot dy
\]

### Fluid drag

\[
F_w = c_w(\text{Re}(u_c)) \cdot \rho_f \cdot A_p \cdot \frac{u_c^2(t)}{2}
\]

\[
\Delta p = \frac{F_w}{A} = \text{Eu}(\text{Re}(u_c)) \cdot \rho_f \cdot \frac{u_c^2(t)}{2}
\]

### Inertia

\[
F_T = \rho_s \cdot V_p \cdot \dot{v}(t)
\]

\[
\frac{F_T}{A} = \rho_{tr} \cdot \dot{v}(t) \cdot dy
\]
Prof. Dr. J. Tomas, chair of Mechanical Process Engineering

**Fluid Flow through Particle Beds**

1. **Darcy’s law** (development of water purification process, model: laminar permeation of groundwater through sand, \(Re < 0.5 \ldots 20\)):
   
   \[ u \propto \text{grad} \, p \Rightarrow u = k \cdot \text{grad} \, p \quad \text{or} \quad \dot{V} = k \cdot A \cdot \text{grad} \, p \]  

   (1)  
   (2)  

   **original Darcy (1856):**

   \[ \text{grad} \, p = \frac{\Delta h_w}{\Delta h_b} \]  

   (3)

   \[ \dot{V} = k_f \cdot A \cdot \frac{\Delta h_w}{\Delta h_b} \]  

   (4a)

   or \[ u = k_f \cdot \frac{\Delta h_w}{\Delta h_b} \]  

   (4b)

   \(k_f\) – permeability

2. **Permeability according to Carman and Kozeny:**

   \[ u = \frac{\varepsilon^3}{K_{CK} \cdot \eta \cdot A_{S,V}^2 \cdot (1 - \varepsilon)^2} \cdot \text{grad} \, p \]  

   (5)

   \[ k_f = \frac{\varepsilon^3 \cdot \rho_f \cdot g}{K_{CK} \cdot \eta \cdot A_{S,V}^2 \cdot (1 - \varepsilon)^2} = \frac{\varepsilon^3 \cdot \rho_f \cdot g \cdot d_{ST}^2}{36 \cdot K_{CK} \cdot \eta \cdot (1 - \varepsilon)^2} \]  

   (6)

3. **Reference values of permeability and flow behaviour (flow function \(ff_c\)):**

<table>
<thead>
<tr>
<th>(k_f^{1)}) in m/s</th>
<th>permeability</th>
<th>soil behaviour</th>
<th>(ff_c = \sigma_f / \sigma_c)</th>
<th>flowability</th>
<th>(\approx d_{ST}^{2)}) in (\mu m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 10^{-9}</td>
<td>practically impermeable (- 3.15 cm/a) very low (- 26 cm/mon)</td>
<td>very binding</td>
<td>0 - 2</td>
<td>very cohesive</td>
<td>0 - 0.5</td>
</tr>
<tr>
<td>10^{-9} - 10^{-7}</td>
<td>low (- 86 cm/d)</td>
<td>low binding</td>
<td>2 - 4</td>
<td>cohesive</td>
<td>0.5 - 5</td>
</tr>
<tr>
<td>10^{-7} - 10^{-5}</td>
<td>medium (- 3.6 m/h) high</td>
<td>non binding</td>
<td>&gt; 4</td>
<td>easy to free flowing</td>
<td>50 - 500</td>
</tr>
<tr>
<td>10^{-5} - 10^{-3}</td>
<td>10^{-3} - 1</td>
<td></td>
<td></td>
<td></td>
<td>500 - 15 mm</td>
</tr>
</tbody>
</table>

1) according to Terzaghi / Peck

2) \(K_{CK} = 5\) (spheres), \(\rho_f = 10^3 \text{ kg/m}^3\), \(\eta = 10^{-3} \text{ Pa.s}\), \(\varepsilon = 0.38\)
## Stressing and Flow of Wet Particle Dispersions

<table>
<thead>
<tr>
<th>Suspension and Particle Flow Pattern</th>
<th>Particle in Liquid Dispersion (Suspension)</th>
<th>Paste</th>
<th>Liquid in Particle Packing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diluted</td>
<td>u_x</td>
<td>v_x</td>
<td></td>
</tr>
<tr>
<td>Concentrated</td>
<td>u_x</td>
<td>v_x</td>
<td></td>
</tr>
<tr>
<td>Liquid Saturated</td>
<td>u_x</td>
<td>v_x</td>
<td></td>
</tr>
<tr>
<td>Moist Packing</td>
<td>u_x</td>
<td>v_x</td>
<td></td>
</tr>
</tbody>
</table>

### Flow Function
- \( \tau \propto f(\sigma) \)
- \( \tau \propto f(\dot{\gamma}) \)

### Cubical Cell Packing Model
- \( \frac{\phi_s}{\varepsilon_{s,0}} = (1 + \frac{a}{d})^3 \)

### Particle Separation
- \( \frac{a}{d} > 1 \)
- \( 0 < \frac{a}{d} < 0.2 \)
- \( \frac{a}{d} = 0 \)
- \( -0.01 < \frac{a}{d} < 0 \)

### Particle Volume Fraction
- \( \phi_s < 0.066 \)
- \( 0.3 < \phi_s < \frac{\pi}{6} \)
- \( \varepsilon_{s,0} = \frac{\pi}{6} \)
- \( \varepsilon_s > \frac{\pi}{6} \)
- Pore saturation \( S = 1 \)
- \( S < 1 \)

### Particle Friction
- \( \phi_f = 0 \)
- \( \phi_f = 0 \)
- \( \phi_f \geq 0 \)
- \( \phi_f > 30^\circ \)
Vortex Flow of Fluids

a) Rigid body eddy

\[ \omega = \text{const.} \]
\[ \frac{u_\varphi(r)}{r} = \text{const.} \]

b) Potential eddy (vortex flow)
frictionless, isoenergetic

\[ p(r) \sim \frac{1}{r^2} \cdot \text{const.} \]
\[ u_\varphi(r) \sim \frac{1}{r} \]
\[ \text{rot } u_\varphi = 2 \cdot \omega \]

\[ u_\varphi(r) \cdot r = \text{const.} \]
\[ p(r) = p_\infty - \frac{\rho}{2} u_\varphi^2(r) \]

\[ r(\varphi) = a \cdot \exp(\tan \beta \cdot \varphi) \]

\[ \frac{u_r(r)}{u_\varphi(r)} = \text{const.} = \tan \beta \]

sink stream

\[ u_r(r) \cdot r = -\text{const.} \]
a) Turbulent flow rate fluctuations

- Local fixed (Euler)
- Formula: \( u(x, y, t) \)

b) Macroturbulence

- Moving coordinates (Lagrange)
- Formula: \( \text{Re}_W = \frac{u^* r_W}{v} \)
- Conditions:
  - \( \text{Re}_W < 30 - 40 \) laminar eddy
  - \( \text{Re}_W > 30 - 40 \) turbulent eddy
  - \( \text{macrodimension} \Lambda = 4 r_W \)

- Turbulence intensity: \( T_u = \sqrt{\frac{u'^2}{u}} \)

- Turbulent eddy viscosity: \( \nu_t = 0.09 \times \frac{k^2}{\varepsilon} = 0.09 \frac{(u_x'^2 + u_y'^2 + u_z'^2)^2}{dP/dm} \)

- Dissipation rate: \( \varepsilon = 1.65 \times \left( \frac{u'^2}{\Lambda} \right)^{3/2} \)

- Kolmogorov dimension: \( l_D = \left( \frac{v^3}{\varepsilon} \right)^{1/4} \)
Thermokinetic Particle Diffusion in Dispersion Medium

Prerequisites:
- External field forces = 0
- Fluid drag = 0
- Fluid flow rates \( u_x = u_y = u_z = 0 \)
- One-dimensional model \( \frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0 \)

a) Vessel with separation membrane

Boundary and initial conditions:
- For \( t = 0, x < 0 \), \( c_p = c_{p,0} \)
- For \( t = 0, x > 0 \), \( c_p = c_{p,1} = 0 \)
- For \( t = \infty \), \( c_{p,E} = \frac{(c_{p,0} + c_{p,1})}{2} \)

b) Time and spatial function of the particle concentration \( c_p \)

\[
c_p(x, t) = c_{p,0} - (c_{p,0} - c_{p,1}) \cdot \phi(x, t)
\]

\[
\phi(x, t) = \frac{1}{\sqrt{2\pi} D_p} \exp\left(-\frac{x^2}{4D_p t}\right)
\]

\( \phi(x < 0, t = 0) = 0 \)

\( \phi(x = 0, t) = \phi(x, t = \infty) = 0.5 \)

\( \phi(x = \infty, t) = 1 \)

Characteristic diffusion distance or standard deviation

\[
\sigma_x = x_1 - x_0 = \sqrt{2 \cdot D_p \cdot t}
\]

\( \sigma_x = \frac{c_{p,0} \cdot c_{p,1}}{2} \)

\[
D_p = \frac{(c_{p,0} - c_{p,1})^2}{4\pi t \cdot (\delta c_p / \delta x)_{max}}
\]

c) Time and spatial function of the particle concentration gradient \( \text{grad} c_p \approx \frac{\delta c_p}{\delta x} \)

d) Time and spatial function of the slope of particle concentration gradient

\[
div (\text{grad} c_p) = \frac{\delta^2 c_p}{\delta x^2}
\]

\[
\frac{\delta^2 c_p}{\delta t \delta x^2}
\]
Particle Transport in Turbulent Fluid Flow

a) Schematic graph of turbulent transport by eddies

b) To derive the transport equation of particles (2-dimensional plane flow)

c) Particle concentration in a homogeneous turbulence field

| characteristic equilibrium state | concentration distribution | 1. Sedimentation in non-turbulent suspension $D_{tz} = 0, v_s > 0$
|----------------------------------|---------------------------| 2. Sedimentation of coarse or heavy particles in turbulent suspension $D_{tz} > 0, v_s >> 0$
| ![Graph 1](image1) | $c_n = \exp \left(-z \frac{v_s}{D_{tz}}\right)$ | 1. $v_s = 0$, if $\rho_s = \rho_f$
| ![Graph 2](image2) | ![Graph 3](image3) | 2. $v_s \to 0$, if $d \to 0$
| ![Graph 4](image4) | ![Graph 5](image5) | 3. high turbulence intensity
| ![Graph 6](image6) | ![Graph 7](image7) | 1. Moderate turbulence intensity $D_{tz} > 0$
| ![Graph 8](image8) | ![Graph 9](image9) | 2. wide distribution of particle settling velocity $v_s > 0$
| ![Graph 10](image10) | ![Graph 11](image11) | 3. Exponential height distribution of particles

Fig. MPE_4 © Prof. Dr. J. Tomas Mechanical Process Engineering - Particle Technology 04.12.2012
# Turbulent Particle Separation Apparatuses

<table>
<thead>
<tr>
<th>1. Cross flow separation apparatus</th>
<th>REYNOLDS number ( Re = \frac{u \cdot D}{v} )</th>
<th>degree of turbulence ( Tu = \sqrt{\frac{u^2}{D}} )</th>
<th>turb. diffusion coefficient in ((cm)^2/s)</th>
<th>BODENSTEIN number ( Bo = \frac{v \cdot L}{D_t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 screw classifier</td>
<td>( \frac{n_s \cdot D_s^2}{v} )</td>
<td>0.05 – 0.15</td>
<td>( 0.014 \cdot n_s \cdot D_s^2 + \frac{0.48 \cdot V_f}{B} )</td>
<td>( \frac{n_s \cdot D_s^2}{D_t} \approx 100 )</td>
</tr>
<tr>
<td></td>
<td>( Re_{crit} \approx 10^4 )</td>
<td></td>
<td>5 - 50</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 10^4 - 5 \cdot 10^5 )</td>
<td></td>
<td>( \approx (2)^2 - (7)^2 )</td>
<td></td>
</tr>
<tr>
<td>1.2 rake classifier</td>
<td>( \frac{n_R \cdot L_R^2}{v} )</td>
<td>-</td>
<td>( 0.31 \cdot n_R \cdot L_R^2 + \frac{0.48 \cdot V_f}{B} )</td>
<td>( \frac{n_R \cdot L_R^2}{D_t} \approx 1.5 - 3 )</td>
</tr>
<tr>
<td></td>
<td>( 10^4 - 5 \cdot 10^4 )</td>
<td></td>
<td>30 - 100</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( \approx (5.5)^2 - (10)^2 )</td>
<td></td>
</tr>
<tr>
<td>1.3 cyclones</td>
<td>( \frac{u \cdot D_c}{v} )</td>
<td>0.01 - 0.05</td>
<td>water:</td>
<td>( \frac{u \cdot D_C}{D_t} \approx 10^3 )</td>
</tr>
<tr>
<td></td>
<td>( Re_{crit} \approx 10^3 )</td>
<td>( \approx 0.1 ) at input</td>
<td>8 \cdot 10^{-4} \cdot u \cdot D_c</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 10^5 - 10^6 )</td>
<td></td>
<td>air: 0.0035 \cdot u \cdot D_c</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 - 20</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( \approx (1)^2 - (4.5)^2 )</td>
<td></td>
</tr>
<tr>
<td>1.4 zigzag apparatus*</td>
<td>( \frac{u \cdot b}{v} )</td>
<td>( Tu \approx \frac{D_t}{u \cdot b} \approx 0.11 - 0.13 )</td>
<td>(0.11 – 0.13) \cdot u \cdot b ( 2000 - 4000 )</td>
<td>( \frac{u \cdot b}{D_t} \approx 1 - 15 )</td>
</tr>
<tr>
<td></td>
<td>( 10^4 - 6 \cdot 10^5 )</td>
<td></td>
<td>( \approx (45)^2 - (63)^2 )</td>
<td></td>
</tr>
<tr>
<td>2. counter-current separation apparatus</td>
<td>( \frac{u \cdot D}{v} )</td>
<td>-</td>
<td>( 0.02 \cdot u \cdot D )</td>
<td>( 0.5 - 50 )</td>
</tr>
<tr>
<td>2.1 counter-current classifier</td>
<td>( 10^3 - 10^6 )</td>
<td></td>
<td>200 - 2000</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( \approx (14)^2 - (45)^2 )</td>
<td></td>
</tr>
</tbody>
</table>

In Schubert, H.: Aufbereitung fester mineralischer Rohstoffe, Verlag für Grundstoffindustrie Leipzig 1989

* Back-calculated from separation tests
### Selected Flow Separation Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Separation Function</th>
<th>Cut Size</th>
<th>Separation Efficiency</th>
<th>Rem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>KAISER 1963</td>
<td>$T(d) = \frac{1}{1 + \left(\frac{1}{T_z} - 1\right)^z}$</td>
<td>-</td>
<td>$\kappa = \frac{d_{25}}{d_{75}} = \text{or} \ T' = $</td>
<td>CCS z</td>
</tr>
<tr>
<td>MOLERUS 1967/69</td>
<td>$\frac{1}{1 + \left(\frac{u}{v_s(d)}\cdot \exp\left(-\frac{(v_s(d) - u)\cdot H}{D_{ax}}\right)\right)}$</td>
<td>$d_{50} = d_T(T = 0,5) = $</td>
<td>$\kappa \approx \frac{1}{z}$</td>
<td>CCS</td>
</tr>
<tr>
<td>NEESSE SCHUBERT 1969/73</td>
<td>$1 - \exp\left(-\frac{v_s(d)\cdot H}{D_t}\right)$</td>
<td>$\kappa = \frac{d_{25}}{d_{75}} = \text{or} \ T' = $</td>
<td>-</td>
<td>CFS</td>
</tr>
<tr>
<td>SCHUBERT, NEESSE 1973 tapping</td>
<td>$\frac{1}{1 + \frac{V_F}{V_G}\cdot \exp\left(-\frac{v_s(d)\cdot H}{D_t}\right)}$</td>
<td>$\kappa = \frac{d_{25}}{d_{75}} = \text{or} \ T' = $</td>
<td>-</td>
<td>CFS</td>
</tr>
<tr>
<td>SENDEN 1979</td>
<td>$T_{L,0} = \frac{1 - p_A + K}{p_{S(0,0)} \cdot \left(\frac{p_L}{1 - p_S}\right)^A + \left(1 - \frac{p_L}{1 - p_S}\right) \cdot \left(1 - \frac{p_L}{1 - p_S}\right)} + K$</td>
<td>-</td>
<td>-</td>
<td>MP z</td>
</tr>
<tr>
<td>BÖHME 1986</td>
<td>$\frac{1}{1 + \frac{u - v_s}{k_F\cdot u}\cdot \exp\left(-\frac{(u - v_s)\cdot H_G}{D_t}\right) - 1}$</td>
<td>$\kappa = \frac{d_{25}}{d_{75}} = \text{or} \ T' = $</td>
<td>-</td>
<td>CCS</td>
</tr>
<tr>
<td>HUSEMANN 1990</td>
<td>$\frac{1}{1 + \frac{\dot{m}<em>0\cdot (u - v_s) + 1}{k \cdot v_s \cdot m_A} \cdot \exp\left(-\frac{(u - v_s)\cdot R_G\cdot (R_s - R_G)}{(u + v_s)\cdot R_s\cdot (s</em>{SS} + a_{SS})}\right) - 1}$</td>
<td>-</td>
<td>-</td>
<td>CCS</td>
</tr>
</tbody>
</table>

CFS cross flow separation; $z$ number of separation stages; CCS counter-current separation; MP MARKOFF process; $a$ acceleration; $\alpha = 2$ STOKES; $\alpha = 0,5$ NEWTON
1. Operation principles of particle flow classification
   a) Cross-flow air classification (horizontal flow separator)

   b) Laminar cross-flow hydroclassification

   c) Turbulent cross-flow hydroclassification

2. Separation model of laminar cross-flow hydroclassification

3. Particle number concentration $c_{n,i}$ of size fraction $i$ versus apparatus height $H$ at counter-current classification

   Height coordinate
   - $H_2$ discharge of fines
   - $y$
   - $\pm 0$ feed
   - $H_1$ discharge of coarse

concentration steps at steady-state operation
Prerequisites for Turbulent Cross Flow Separation Model

(1) **Particle hold-up probability distribution** (concentration per number \( c_{n,i,j} \)) versus height \( y \) independent each other, i.e., for every particle size fraction \( i \) as well as density fraction \( j \) the FOKKER-PLANCK Eq. is valid:

\[
\frac{\partial c_{n,i,j}}{\partial t} = -(-v_{s,i,j}) \cdot \frac{1}{1!} \frac{\partial c_{n,i,j}}{\partial y} + D_{t,s} \cdot \frac{1}{2!} \frac{\partial^2 c_{n,i,j}}{\partial y^2} - ... + ...
\]

(1)

(2) For a homogeneous field of turbulence in the process chamber turbulent diffusion coefficient \( D_t \approx D_{t,s} \) particle diffusion coefficient, i.e. turbulence intensification by free turbulent particle flow pattern > turbulence damping by particle concentration

\[
\Lambda \cdot \sqrt{u_x^2} \approx \text{const.} = D_t
\]

(2)

(3) Macro dimension of turbulence (diameter of largest eddies \( d_{W,max} = \Lambda/2 \)), \( \equiv \) characteristic dimension of a turbulence generating tool, here channel width \( b \approx 0.2 \text{ m} \),

\[
\Lambda \propto b
\]

(3)

(4) The root mean square (RMS) of turbulent flow rate fluctuations across principal flow direction \( \propto \) eddy circumferential speed \( u_\phi \equiv \text{charact. flow rate, channel flow rate averaged} \ u \)

\[
\sqrt{u_x^2} \propto u_\phi \propto \bar{u}
\]

(4)

(5) Particle size small compared with macro dimension of turbulence, i.e. channel width

\[
d < 0.1 \cdot \Lambda < b
\]

(5)

(6) Particle size small compared with micro dimension of turbulence (\( d_{W,min} \) diameter of smallest eddys with circular laminar flow), here not valid

\[
d < d_{W,min} \approx 10 \cdot l_D \approx 0.3 \text{ mm}
\]

(6)

**KOLMOGOROV dimension**

\[
l_D = \left(\frac{\nu^3}{\varepsilon}\right)^{\frac{1}{4}} = \left(15^3 \cdot 10^{-18} / 4W / g\right)^{\frac{1}{4}} \approx 30 \mu m
\]

(7)

(7) For steady-state condition \( \frac{\partial c_{n,i,j}}{\partial t} = 0 \) (at bottom \( y = 0 \), \( c_{n,i,j} = c_{n,0,i,j} \)) an exponential particle concentration distribution versus height \( h \) is valid

\[
\frac{c_{n,i,j}}{c_{n,0,i,j}} = \exp\left(-\frac{v_{s,i,j}}{D_{t,s}} \cdot h\right)
\]

(7)
4. Separation models of turbulent cross-flow hydroclassification (Neeße/Schubert)

a) Suspension splitting model

\[ \dot{V} = \dot{V}_F + \dot{V}_G \]

\[ H_G = \frac{V_G}{\dot{V}} \]

\[ T_i = \frac{c_{n,G,i} \cdot H_G}{c_{n,A,i} \cdot H} \]

\[ = \frac{1 - \exp \left( -\frac{v_{s,i}}{D_{td}} H_G \right)}{1 - \exp \left( -\frac{v_{s,i}}{D_{td}} H \right)} \]

\[ \dot{V}, \dot{V}_G, \dot{V}_F \] suspension volume flow rate of feed, coarse and fine particles

b) Suspension tapping model

\[ \dot{V} = \dot{V}_F + \dot{V}_G \]

\[ H_G = \frac{V_G}{\dot{V}} \]

\[ T_i = \frac{c_{n,0,i} \cdot \dot{V}_G}{c_{n,0,i} \cdot \dot{V}_G + c_{n,H,i} \cdot \dot{V}_F} \]

\[ = \frac{1}{1 + \frac{\dot{V}_F}{\dot{V}_G} \exp \left( -\frac{v_{s,i}}{D_{td}} H \right)} \]

\[ d_{r} = \sqrt{\frac{1}{k_{\psi} k_{\psi}} \frac{18 \eta D_{td}}{H}} \ln \frac{\dot{V}_F}{\dot{V}_G} \]

\[ \kappa = \frac{d_{25}}{d_{75}} = \left( \frac{\ln \left( \dot{V}_F / \dot{V}_G \right) - \ln 3}{\ln \left( \dot{V}_F / \dot{V}_G \right) + \ln 3} \right)^{0.5} \]

5. Normalized separation function

\[ T(d/d_{r}) = \frac{1}{1 + \left( \frac{\dot{V}_F}{\dot{V}_G} \right)^{d/d_{r}}} \]

a) for \( \alpha = 2 \) (Stokes range) and different ratios of volume flow rate

b) for different \( \alpha \) -values at \( \dot{V}_F / \dot{V}_G = 4 \)

6. Particle segregation of dilute and dense flow at cross-flow hydroclassification:

a) dilute flow segr.

\[ \varphi_s < 5 - 10 \% \]

b) dense flow segr.

\[ \varphi_s > 30 \% \]

c) combined dilute/dense flow

\[ \varphi_s \approx 10 - 30 \% \]
Model of Counter-current Particle Separation

Particle number concentration profile in the process chamber:

<table>
<thead>
<tr>
<th></th>
<th>Coarse (G)</th>
<th>Equilibrium particle</th>
<th>Fines (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle absolute velocity</td>
<td>$v_s &gt; u$</td>
<td>$v_{s,T} = u$</td>
<td>$v_s &lt; u$</td>
</tr>
<tr>
<td>$v_{a,G} &lt; 0$</td>
<td>$v_{a,G} &lt; 0$</td>
<td>$v_{a,F} &gt; 0$</td>
<td>$v_{a,F} &gt; 0$</td>
</tr>
</tbody>
</table>

0. Feed

$y = 0$

$c_{n,0} = c_{n,1} \left[ 1 + \frac{k_{1} \cdot u_{G}}{D_{t,s}} \cdot y_{1} \right]$

I. Underflow

$y_{1} < y < 0$

$c_{n,1} = \frac{c_{n,1}}{v_{a}} \cdot \left\{ k_{1} \cdot u_{G} + (v_{s} + k_{1} \cdot u_{G}) \cdot \exp \left[ \frac{V_{G}}{D_{t,s}} \cdot (y + y_{1}) \right] \right\}$

$c_{n,T} = c_{n} \left[ 1 + \frac{k_{1} \cdot u_{G}}{D_{t,s}} \cdot (y + y_{1}) \right]$
Turbulent Counter-current Separation Model

(1) Particle absolute velocity \( v_a(d) \) in locally fixed coordinates of the apparatus:

\[
\tilde{v}_a(d) = \tilde{u} - \tilde{v}_s(d)
\]  

(1)

(2) Separation function:

\[
T(v_a(d)) = \frac{1}{1 + \left(1 + \frac{v_{a,l}}{k_1 \cdot u_G} \right) \cdot \exp \left[ \frac{v_{a,l}}{D_{t,s}} \cdot y_1 \right]}
- \frac{1 - \left(1 + \frac{v_{a,ll}}{k_2 \cdot u} \right) \cdot \exp \left[ -\frac{v_{a,ll}}{D_{t,s}} \cdot y_2 \right]}{1 + \left(1 + \frac{v_{a,ll}}{k_2 \cdot u} \right) \cdot \exp \left[ -\frac{v_{a,ll}}{D_{t,s}} \cdot y_2 \right]}
\]

(2)

(3) Mean residence time:

\[
\tau_m = \frac{1}{\hat{n}_A} \cdot \int_{y_1}^{y_2} c_a(y) \, dy
\]

\[
\tau_m = \frac{1}{v_a} \cdot \left[ T \cdot \left( y_1 - \frac{D_{t,s}}{k_1 \cdot u_G} \right) + (1 - T) \cdot \left( y_2 + \frac{D_{t,s}}{k_2 \cdot u} \right) \right]
\]

(3)

(4) Incremental separation sharpness (slope for \( d \to d_T \)) instead using \( \kappa \):

\[
\frac{d[T(d/d_T)]}{d(d/d_T)} \bigg|_{d \to d_T} = \frac{\alpha \cdot u \cdot H}{4 \cdot D_{t,s}} \cdot \left( 1 + \frac{1}{1 + k \cdot \frac{u \cdot H}{D_{t,s}}} \right)
\]

(4)

for large BODENSTEIN numbers (mainly convective transport):

\[
Bo = \frac{u \cdot H}{D_{t,s}} >> 1
\]

(5)

\[
\frac{d[T(d/d_T)]}{d(d/d_T)} = \frac{\alpha}{4} \cdot Bo \cdot \left( 1 + \frac{1}{1 + k \cdot Bo} \right) \approx \frac{\alpha}{4} \cdot Bo
\]

(6)
Prof. Dr. J. Tomas, chair of Mechanical Process Engineering

Evaluation of Turbulent Counter-current Hydroclassification

1. Separation function $T(v_s(d))$ and medium residence time $\tau_m(v_s(d))$ versus stationary settling velocity $v_s(d)$ for $k_1 = k_2 = 1$; $H_1 = H_2 = 1$ m
   a) Different counter-flow rates $u$ for BODENSTEIN-number $Bo = u H/D_t = 10$

   ![Graph 1.1](image1.png)

   b) Different BODENSTEIN-numbers $Bo = u H/D_t$ for $u = 0.5$ m/s and $H = 1$ m

   ![Graph 1.2](image2.png)

2. Separation function $T(v_s(d))$ versus stationary settling velocity $v_s(d)$ for $u = 0.5$ m/s; $H = 1$ m; $Bo = 10$
   a) Different heights of separation chamber for $k_1 = k_2 = 1$
   b) Different discharge coefficients for $H_1 = H_2 = 1$ m
   c) Different height ratios $H_1/H_2$ of separation sub-chambers for $H_1 + H_2 = 2.5$ m and $k_1 = k_2 = 1$

   ![Graph 2.1](image3.png)

   ![Graph 2.2](image4.png)

   ![Graph 2.3](image5.png)
**Principles of Particle Separation in an Air Stream (Sifting)**

1. **Counter-current separation (sifting)**
   a) *in a gravity field*
   
   ![Diagram](image1)

   \[ \begin{align*}
   & v_s < u \\
   & v_s = u \\
   & v_s > u
   \end{align*} \]

   \[ v_s < u \quad v_s > 0 \quad v_s = 0 \quad v_s < u \]

   - [Diagram](image2)

2. **Cross-flow separation (sifting with gravitational force and force of inertia)**
   a) *horizontal cross-flow sifter (separator)*
   
   ![Diagram](image3)

   b) *cross-flow u-turn sifter*
   
   ![Diagram](image4)

   c) *vertical cross-flow sifter*
   d) *cross-flow jet sifter*
   e) *blind u-turn sifter*
Particle Separation and Classification

FIG. 4.31  Particle-size range as a guide to the range of applications of various solid-solid operations.
<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
<th>Max (μm) Width</th>
<th>Max. N (mm)</th>
<th>Limiting size (μm)</th>
<th>Fixed mass (%)</th>
<th>Fixed mass (%)</th>
<th>Power (kW)</th>
<th>Stability and applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slotted drum classifier</td>
<td>Classification occurs near the end of the drum, avoiding pool</td>
<td>0.09 to 0.4</td>
<td>2.4 long</td>
<td>0.09 to 0.20</td>
<td>0.8 mm</td>
<td>0.20 to 0.30</td>
<td>0.8 to 1.1</td>
<td>Used for closed-circuit grinding, washing, and desizing. Easily modifiable, especially when the drum is moved, allowing for easy change of drum position.</td>
</tr>
<tr>
<td>Logwasher</td>
<td>Essentially a spiral classifier with paddles replacing the spiral</td>
<td>0.06 to 0.9</td>
<td>0.09 to 1.5</td>
<td>0.06 to 1</td>
<td>0.90 mm</td>
<td>0.20 to 0.30</td>
<td>0.90 to 2.6</td>
<td>Used for rough separations such as removing trash, dirt, and other debris from mixtures.</td>
</tr>
<tr>
<td>Roll classifier</td>
<td>Evolution of the drum classifier, using multiple rolls operating in parallel</td>
<td>0.06 to 0.9</td>
<td>1.2 to 10</td>
<td>0.20 to 0.30</td>
<td>1.00 mm</td>
<td>0.20 to 0.30</td>
<td>1.00 to 2.6</td>
<td>Used for closed-circuit grinding, partly similar to drum classifiers. Larger roll diameter separation. Roll Diameter has higher pressure and capacity, leading to easier operation.</td>
</tr>
<tr>
<td>Water-screw classifier</td>
<td>Basically a hydraulic type classifier, utilizing a screw conveyor mechanism to push the material through a series of rollers or disks.</td>
<td>0.06 to 0.9</td>
<td>1.2 to 4.5</td>
<td>0.20 to 0.30</td>
<td>1.00 mm</td>
<td>0.20 to 0.30</td>
<td>1.00 to 2.6</td>
<td>Used for closed-circuit grinding, washing, and desizing. Easily modifiable, especially when the drum is moved, allowing for easy change of drum position.</td>
</tr>
<tr>
<td>Cylindrical classifier</td>
<td>Effectively an overhauled, pitchless, rotating classifier that needs no central support.</td>
<td>0.06 to 0.9</td>
<td>1.2 to 4.5</td>
<td>0.20 to 0.30</td>
<td>1.00 mm</td>
<td>0.20 to 0.30</td>
<td>1.00 to 2.6</td>
<td>Used for closed-circuit grinding, washing, and desizing. Easily modifiable, especially when the drum is moved, allowing for easy change of drum position.</td>
</tr>
</tbody>
</table>
TABLE 4.33 The Major Types of Classifiers (Continued)

<table>
<thead>
<tr>
<th>Classifier</th>
<th>Description</th>
<th>Type</th>
<th>Min. Length (mm)</th>
<th>Max. Diameter (mm)</th>
<th>Liner Size (min. 60% or max. 90%)</th>
<th>Fixed Mass (kg)</th>
<th>Vol. % solids Fixed moisture downfall</th>
<th>Power kW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrocyclone classifier</td>
<td>Open cylindrical open section with central closed section remaining slowly rotating mechanism</td>
<td>50-300</td>
<td>400 mm to 1000 mm</td>
<td>1 mm or 0.1 mm</td>
<td>10 to 130</td>
<td>Non critical 30 to 50</td>
<td>3 to 7.5</td>
<td>Used primarily in closed circuit grinding to supplement hydrocyclone section</td>
</tr>
<tr>
<td>Hydrocyclone</td>
<td>Described pressure fluid pressure and centrifugal action give high processing forces and discharge</td>
<td>—</td>
<td>0.8 to 1.2</td>
<td>300 mm or 150 mm</td>
<td>m 20 and</td>
<td>4 to 20</td>
<td>30 to 50</td>
<td>Used where solids must be large, dry, such as coarse grinding. Air classifiers can be integrated into grinding mill systems</td>
</tr>
<tr>
<td>Air classifier</td>
<td>Similar to hydrocyclone, but high enough angle to permit impeller to induce recirculation of fluid</td>
<td>5.0 to 7.0</td>
<td>2 mm or 0.5 mm</td>
<td>2 mm or 0.5 mm</td>
<td>m 2300</td>
<td>4 to 900</td>
<td>Used where solids must be large, dry, such as coarse grinding. Air classifiers can be integrated into grinding mill systems</td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 4.33** Prof. Dr. J. Tomas, chair of Mechanical Process Engineering
Prof. Dr. J. Tomas, chair of Mechanical Process Engineering
Flow Classification - Gravitational Force Hydroclassifier
- Horizontal or Cross-Flow Classifiers

1. Horizontal or cross-flow classifier, schematic:
   a) Cone classifier
   b) Multi-chamber cross-flow classifier

2. Cone classifier with float discharge control
   1 Feed pipe of suspension
   2 Inserts (baffles) to calm the feed flow
   3 Internal cone shell
   4 Float
   5 Overflow discharge sill of fine product
   6 Basic cone
   7 Discharge launder of fine product
   8 Lever & rod mechanism
   9 Ball support
   10 Underflow ball valve for coarse product
   11 Adjustable mass
   12 Spring
   13 Discharge obstacle (disk)
Flow Classification - Gravitational Force Hydroclassifier
- Horizontal or Cross-Flow Classifiers

3. Cross-flow classifier with mechanical discharge device of coarse product
   (mechanical classifier), schematic:
   a) Rake classifier
   b) Screw classifier

4. Screw classifier, version SKET

5. Arrangement of horizontal or cross-flow classifiers, version Rheax
   a) Counter-current arrangement
   b) "Phalanx" arrangement
1. Upstream or counter-current classifiers (hydrosizers) of different versions:
   a) Rheax  
   b) Sogreah (Lavoflux)  
   c) Hydrosort (with fluidized bed)

   d) Rheax (zigzag classifier)  
   d) Hydrofors  
   e) Larox
**Fig. 20-41** Typical centrifugal separator.
Screw classifier

Version Wemco S-H 78 in a closed milling circuit of St. Joseph Lead Co., Indian Creek Plant
Counter-current classifier
Version TAK Amberger
Kaolinwerke
for sand, cut size:
above 200 - 500 µm
F  feed
K₁  coarse product
K₂  fine product
Fl  fluid
Counter-current Control in Counter-current Classifiers

F feed, K₁ coarse product, K₂ fine product, Fl fluid
1 pump control by revolution number
2 height adjustable diving cone insert
3 feed valve
4 walls to separate segments
Particle Flow Separation by Hydrocyclone

Schematic view of fluid flow

Oblique or horizontal built-in is possible
Particle Flow Separation by Hydrocyclone

Version TAK Amberger Kaolinwerke

200 mm Zyklon
RWT 4118

1 Überlaufduse
2 Einlassstück
3 Zylinder
4 Konus
5 Konusverlängerung
6 Apex-Stopfen
Multi-cyclones, Cyclone Batteries

- pre-processing or replacement of thickeners
- separation of clay and solid, non-soluble constituents
- thickening/cleaning of suspensions
- multistage cyclone arrangements
- washing by cyclones
Multi-cyclones, Cyclone Batteries

Feed distribution vessel of cyclone battery with wear-resistant lining by ceramics

Feed vessel during assembly