3. Particle separation
   3.1 Process principles of particle separation in particle technology
   3.2 Evaluation of separation efficiency by separation probability (function)
   3.3 Particle separation by sieving
   3.3.1 Fundamentals and microprocesses of sieving
   3.3.2 Model of screening dynamics
   3.3.3 Sieving machines and screens
Particle Separation Principles in Particle Technology

<table>
<thead>
<tr>
<th>Particle separation characteristic</th>
<th>Unit operation</th>
<th>Operation principle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>float and sink cleaning</td>
<td>F</td>
</tr>
<tr>
<td>Optical, colour, shape</td>
<td>channel washing</td>
<td>F</td>
</tr>
<tr>
<td>Radiation emission, reflexion, diffraction, particle shape</td>
<td>semipermeable membrane separation</td>
<td>F, P</td>
</tr>
<tr>
<td>Magnetic susceptibility</td>
<td>magnetic separation</td>
<td>F, P</td>
</tr>
<tr>
<td>Conductivity</td>
<td>electric separation</td>
<td>F, P</td>
</tr>
<tr>
<td>Wetting</td>
<td>flotation</td>
<td>F, P</td>
</tr>
<tr>
<td>Size, shape, molar mass</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

fragment or particle size \(d\) in mm:

- \(10^{-5}\)
- \(10^{-4}\)
- \(10^{-3}\)
- \(0.01\)
- \(0.1\)
- \(1\)
- \(10\)
- \(100\)
- \(10^3\)
Separation (Grading) Efficiency Assessment

- total mass balance
\[ m_A = m_G + m_F \]  
(1)

- component balance
\[ m_A \cdot \mu_{A,i} = m_G \cdot \mu_{G,i} + m_F \cdot \mu_{F,i} \]  
(2)

- recovery coarse product G
\[ R_{m,G} = \frac{m_G}{m_A} \]  
(3)

- recovery of valuable coarse fraction i
\[ R_{m,i} = \frac{m_{G,i}}{m_{A,i}} = \frac{m_G \cdot \mu_{G,i}}{m_A \cdot \mu_{A,i}} \]  
(4)

- grading ratio
\[ A_i = \frac{\mu_{G,i}}{\mu_{A,i}} > 1 \]  
(5)

- separation function of a particle characteristic fraction \( \Delta \xi_i \)
\[ T_i(\xi) = \frac{m_{G,i}}{m_{A,i}} = \frac{m_G \cdot \mu_{G,i}}{m_A \cdot \mu_{A,i}} = R_{m,G} \cdot A_i \]  
(6)

- separation sharpness
\[ \kappa = \frac{\xi_{25}}{\xi_{75}} \leq 1 \]  
(7)

Separation/Classification of particles according to size fractions \( \Delta d_i \):

- perfect separation \( \kappa = 1 \)
- misplaced product \( 0.3 < \kappa < 0.6 \) sufficient
- \( 0.6 < \kappa < 0.8 \) good
- \( 0.8 < \kappa < 0.9 \) very good
Separation Function (Grade Efficiency Curve)

Input mass flow rate $\dot{m}_A$

\[ \dot{m}_F \text{ mass flow rate of fine particles} \]
\[ \dot{m}_G \text{ mass flow rate of coarse particles} \]

\textbf{Mass balance:}

\textbf{Total:} \hspace{1cm} \dot{m}_A = \dot{m}_F + \dot{m}_G \hspace{1cm} (1)

\textbf{Component balance of each fraction } i: \hspace{1cm} \dot{m}_A \cdot \mu_{A,i} = \dot{m}_F \cdot \mu_{F,i} + \dot{m}_G \cdot \mu_{G,i} \hspace{1cm} (2)

with mass fractions $\mu_{A,i} = \frac{\dot{m}_{A,i}}{\dot{m}_{A,tot}}$, $\mu_{F,i} = \frac{\dot{m}_{F,i}}{\dot{m}_{F,tot}}$, $\mu_{G,i} = \frac{\dot{m}_{G,i}}{\dot{m}_{G,tot}}$

Mass recovery of fines $F$ \hspace{1cm} $R_{m,F} = \frac{\dot{m}_F}{\dot{m}_A}$

Mass recovery of coarse $G$ \hspace{1cm} $R_{m,G} = \frac{\dot{m}_G}{\dot{m}_A}$

Total mass balance Eq.(1) can be rewritten as:

\[ l = R_{m,F} + R_{m,G} \hspace{1cm} (1a) \]

and Eq.(2) as \hspace{1cm} $\mu_{A,i} = R_{m,F} \cdot \mu_{F,i} + R_{m,G} \cdot \mu_{G,i} \hspace{1cm} (2a)$

\textbf{Separation function:}

Separation function is defined as the ratio of mass of all particles with size $i$ in the separated coarse product to mass of particles with the same size $i$ in the feed:
Prof. Dr. J. Tomas, chair of Mechanical Process Engineering

\[ T_G(d_i) = \frac{\dot{m}_{G,i}}{\dot{m}_{A,i}} = \frac{\dot{m}_G \cdot \mu_{G,i}}{\dot{m}_A \cdot \mu_{A,i}} = R_{m,G} \cdot \frac{\mu_{G,i}}{\mu_{A,i}} \]  
for coarse product

\[ T_F(d_i) = R_{m,F} \cdot \frac{\mu_{F,i}}{\mu_{A,i}} \]  
for fine product

From Eq.(2a)  \[ T_F(d_i) + T_G(d_i) = 1 \]

The coarse product is used \( T(d_i) = T_G(d_i) \)

To find the separation function we can use Eq.(1a) and Eq.(2a)

It is necessary to know

a) case 1: one of \( R_{m,f} \) or \( R_{m,c} \) and two of three mass fractions \( \mu_{A,i}, \mu_{F,i}, \mu_{G,i} \)

b) case 2: three mass fractions \( \mu_{A,i}, \mu_{F,i}, \mu_{G,i} \)

\[ \kappa = \frac{d_{25}}{d_{75}} \leq 1 \]

\[ 0.3 < \kappa < 0.6 \text{ sufficient} \]
\[ 0.6 < \kappa < 0.8 \text{ good} \]
\[ 0.8 < \kappa < 0.9 \text{ very good} \]

\[ \Rightarrow \text{Separation function characterizes the quality or grade of separation} \]

Additionally one may use:
Prof. Dr. J. Tomas, chair of Mechanical Process Engineering

\[ E_T = \frac{1}{2} \cdot (d_{75} - d_{25}) \quad \text{or} \quad \xi = \frac{1}{2} \cdot \frac{d_{75} - d_{25}}{d_{50}} \]

Ideal or perfect separation:
3.3 Screening

**Screening** (sieving) is the separation of a mixture of various grain sizes into two or more products by a semipermeable membrane.

**Semipermeable membrane** (screening surface) with defined opening size \( w \) acts as a multiple go-no-go gauge.

The final portions consist of grains of more uniform size than those of the original mixture.

**Oversize, overflow, plus** or **coarse** product is that remains on the screen.

**Undersize, underflow, minus** or **fine** product is passing through the openings of screen.

If screen consist of more then one screening surfaces:
One (n) screening or intermediate product is passing through 1\textsuperscript{th} or n\textsuperscript{th} surface and retained on a subsequent surface.

The screening surface may consist of

- woven-wire
- silk
- plastic cloth
- perforated or punched plate
- grizzly bars and wedge wire sections

Aperture or screen-size opening is the minimum free space between the edges of the opening in the screening surface.

Open Area of a screen is the ratio of actual openings versus total screen area $A_{\text{open}}/A_{\text{tot}}$
Sieve Scale in SI-units (mm or µm):
A sieve scale is a series of testing sieves having openings in a fixed succession; for example:
the widths of the successive openings have a constant ratio of, e.g. \( q = \sqrt{2} \) or 1.414, while the areas of the openings have a constant ratio of 2.
Generally it is:
\[
d_n = d_{n-1} q = d_0 q^n
\]
e.g. for \( n = 10 \)
\[
q = \sqrt[10]{2} = \sqrt[10]{1.414} = 1.25
\]
Or for the Tyler scale \( q = \sqrt{2} \) or 1.189.
The Tyler sieve series adopted by the National Bureau of Standards (USA).

The sieve opening is specified in millimeters, which is understood to be the free opening or space between the wires.

Acc. to US-standard wire cloth can be specified by the old non-metric mesh, which is the number of openings per linear inch counting from the center of any wire to a point exactly 25.4 mm (= 1 inch) distance.

- In this non-metric US system mesh is used for cloth \( w < 12.7 \) mm (2 mesh)
- But free openings are used for coarser cloth \( w > 12.7 \) mm
Testing Sieves are generally used for characterization of particle size distribution.

Fundamentals

**Probability screening principle** uses the fact that particles moving almost at right angles to a screening surface have a low probability to pass through when the particle size is larger than about half of the opening.

**Classification of screening operations:**

a) **Scalping** - Strictly, the removing of a small amount of oversize from a feed which is predominantly fines. Typically, the removal of oversize from a feed with approximately a maximum of 5% oversize
   Type of screen: grizzly

b) **Separation** (coarse) - separation at \( w \geq 6 \text{ mm} \) (4 mesh) and larger.
   Type of Screens: Vibrating screen, horizontal or inclined with small angle.
Separation (fine) - separation smaller than $w < 6$ mm (4 mesh) and larger than $w \geq 0.6$ mm (48 mesh) …. 1 mm

Type of Screens: Vibrating screen, horizontal or inclined; high-frequency and low-amplitude vibrating screens.

Separation (very fine) - separation smaller than 0.6 …. 1 mm.

Type of Screens: High-frequency low-amplitude electrically vibrating screens;
⇒ separation in a fluid flow (hydrocyclones, centrifugal wheel separators, wind shifters.

Dewatering - Removal of free water from a solids-water mixture:

Moisture influence on screening efficiency

![Graph showing efficiency vs moisture content]
Screening number (Froude number)

\[ k = \frac{a}{g} \]

Throw number

\[ k_v = \frac{a_{S,\text{max}}}{g \cos \beta} \]

with \( a_{S,\text{max}} \) maximum particle acceleration perpendicular to screen surface 
\( \beta \) inclination angle of screen surface 
\( g \) gravitational acceleration

<table>
<thead>
<tr>
<th>( k_v )</th>
<th>Particle movement</th>
<th>Characterisation of screening behaviour</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \leq 1 )</td>
<td>no throw, planing</td>
<td>very low screening efficiency, obstruction in short time</td>
</tr>
<tr>
<td>1 .. 1,5</td>
<td>no throw of particle layer</td>
<td>slow screening, obstruction</td>
</tr>
<tr>
<td>1,6 .. 1,8</td>
<td>very low throw</td>
<td>very resting screening, easy screen able particle</td>
</tr>
<tr>
<td>2,1 .. 2,3</td>
<td>between low and steep throw</td>
<td>resting screening, difficult screen able particles</td>
</tr>
<tr>
<td>3,0 .. 3,5</td>
<td>steeper throw</td>
<td>Sharp screening, difficult screen able fine particles</td>
</tr>
<tr>
<td>3,3 .. 4,0 .. 8,0</td>
<td>very steep throw</td>
<td>very sharp and scarified screening</td>
</tr>
</tbody>
</table>
Transport of bulk along the sieve

Optimal length of sieve → \( L = v \cdot \tau \)

a) intensive horizontal mixing \( (D_{\text{hor}} \to \infty) \)

b) ideally displacement along sieve \( (D_{\text{ax}} = 0) \)

Model is applicable only at \( h \ll L \)

\[
v = \sqrt{2 \cdot g \cdot h}
\]

and \( h \sim w \)

\[
v = \sqrt{2 \cdot g \cdot w}
\]

- mass flow rate through one opening:

\[
\dot{m}_w = \rho \cdot w^2 \cdot v = \rho \cdot w^2 \cdot \sqrt{2 \cdot g \cdot w}
\]

- mass flow rate through surface that includes one opening and the wire \( (c=0.5) \)

\[
\dot{m}_s = c \cdot \rho \cdot \left( \frac{w-d}{w+d} \right)^2 \cdot \sqrt{2 \cdot g \cdot (w-d)}
\]
**Model of Screening Dynamics**

**Assumption: fast horizontal mixing**

\[ c_G(x, y, z, t) = c_G(t) \rightarrow \text{oversize} \]
\[ c_F(x, y, z, t) = c_F(t) \rightarrow \text{undersize} \]

The probability of passing undersized particles through the sieve

\[ P_k = \left( \frac{w - d}{w + s} \right)^2 \]

The amount of oversized and undersized particles determines the random particle flow (diffusion) through the particle bed on the screen

\[ \frac{d m_F}{d t} = -k_{Gr} \cdot P_k \cdot A \cdot \frac{m_F}{m_G} \]

- **A**: area of screening surface
- **k**: sieve constant for separation \((w / d \geq 10)\)

Assumption \(\rightarrow k(n, p, w, d, P_d) = k(n, p)\)

\[ m_F = m_{F0} \exp \left( -\frac{k_{Gr} \cdot P_k \cdot A \cdot t}{m_G} \right) = m_{F0} \exp \left( -\frac{t}{t_0} \right) \]

with \(t_0 = \frac{m_G}{k_{Gr} \cdot P_k \cdot A} = \frac{1}{k_{\text{sieve}}}\)

What time is necessary for sieving?

Residence time function \(F(t)\) is

\[ F(t) = \frac{m_{F0} - m_F(t)}{m_{F0}} = 1 - \exp \left( -\frac{t}{t_0} \right) \]

- **m_{F0}**: input mass
- **m_F**: the mass in the equipment at the time \(t\)
Residence time distribution is represented by \( f(t) \)

\[
  f(t) = \frac{dF(t)}{dt} = \frac{1}{t_0} \exp\left( -\frac{t}{t_0} \right)
\]

Mean residence time:

\[
  \tau = \int_0^\infty f(t) t dt = \frac{1}{t_0} \cdot (t_0)^2 \int_0^\infty \exp\left( -\frac{t}{t_0} \right) t dt
\]

\[
  = t_0 = \frac{m_G}{k_{Gr} \cdot P_k \cdot A}
\]
Screening machines may be divided into five main classes: grizzlies, revolving screens, shaking screens, vibrating screens, and oscillating screens.

Grizzlies are used primarily for scalping at 50 mm and coarser

Revolving screens and shaking screens are generally used for separations above 13 mm

Vibrating screens cover this coarse range and also down into the fine openings.

Oscillating screens are confined in general to the finer openings below 6 mm.

Grizzly Screens consist of a set of parallel bars held apart by spacers at some predetermined opening. Bars are frequently made of manganese steel to reduce wear. A grizzly is widely used before a primary crusher in rock- or ore-crushing plants to remove the fines before the ore or rock enters the crusher. It can be a stationary set of bars or a vibrating screen.

- stationary grizzly
- flat grizzlies
- vibrating grizzlies.

Revolving screens or drum (trommel) screens, once widely used, are being largely replaced by vibrating screens. They consist of a cylindrical frame surrounded by wire cloth or perforated plate, open at both ends, and inclined at a slight angle. The material to be
screened is delivered at the upper end, and the oversize is discharged at the lower end. Their capacity is not great, and efficiency is relatively low.

**Mechanical Shaking Screens** consist of a rectangular frame which holds wire cloth or perforated plate and is slightly inclined and suspended by loose rods or cables or supported from a base frame by flexible flat springs. The frame is driven with a reciprocating motion. Such devices have given way to vibrating screens. The advantages of this type are low headroom and low power requirement. The disadvantages are the high cost of maintenance of the screen and the supporting structure owing to vibration and low capacity compared with inclined high-speed vibrating screens.

**Vibrating Screens** are used as standard practice when large capacity and high efficiency are desired. The capacity, especially in the finer sizes, is so much greater than that of any of the other screens that they have practically replaced all other types when efficiency of the screen is an important factor. Advantages include accuracy of sizing, increased capacity per unit area, low maintenance cost per ton of material handled, and a saving in installation space and weight.

There are a large number of vibrating screens on the market, but basically they can be divided into two main classes:

- mechanically vibrated screens
- electrically vibrated screens

**Mechanically Vibrated Screens**
Electrically Vibrated Screens are particularly useful in the chemical industry. They handle very successfully many light, fine, dry materials and metal powders from approximately 4 mm (6 mesh) to as fine as 0.078 mm. Most of these screens have an intense, high-frequency (25 to 120 vibrations/s) low-amplitude vibration supplied by means of an electromagnet.
Range of separations that can be obtained with various kinds of screens (to convert inches into millimeters, multiply by 25.4):

![Diagram of screen types and sizes]

- Vibrating screens: Leachey, Hum-mer
- High-speed vibrating screens: Nova, derrick
- Oscillating screens
- Sifter screens: circular, gyratory, circular vibrated motion: Ty-Sifter, Ross, Bar-Nun, Sweco, Rotex
- Centrifugal screen
- V-screen
- Static sieves: Bauer, Wemco, DSM
- Revolving filter screens: North water and sewage screens
- Revolving screens: trommels, scrubbers

(to convert inches into millimeters, multiply by 25.4)